## Acceleration

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## Key Concepts

- Acceleration is a basic measurement in physics that describes the rate of change of velocity of a moving body over time.
- Because acceleration is a directed, or vector, quantity, both direction and magnitude of the moving body should be stated.
- A body will undergo acceleration whenever an unbalanced force acts upon that body.
- Angular acceleration is a vector quantity representing the rate of change of angular velocity of a body experiencing rotational motion.
- Radial acceleration, or centripetal acceleration, is the acceleration of a body toward the center of a circular path.
- Tangential acceleration is the component of linear acceleration tangent to the path of a particle subject to an angular acceleration about the axis of rotation.

The time rate of change of velocity. Acceleration is a physics term that measures the rate of change of velocity per unit of time. Because velocity is a directed or vector quantity involving both magnitude and direction, a velocity may change by a change of magnitude (speed) or by a change of direction, or both. It follows that acceleration is also a directed, or vector, quantity (Fig. 1). See also: acceleration analysis; acceleration MEASUREMENT; ACCELEROMETER; SPEED; VELOCITY.

If the magnitude of the velocity of a body changes from $v_{1} \mathrm{ft} / \mathrm{s}$ to $v_{2} \mathrm{ft} / \mathrm{s}$ in $t$ seconds, then the average acceleration $\bar{a}$ has a magnitude given by equation (1):

$$
\begin{equation*}
\bar{a}=\frac{\text { velocity change }}{\text { elapsed time }}=\frac{v_{2}-v_{1}}{t_{2}-t_{1}}=\frac{\Delta v}{\Delta t} \tag{1}
\end{equation*}
$$

To designate it fully, the direction should be given, as well as the magnitude.


Fig. 1 The direction of the acceleration vector (a) is that of the change in velocity ( $\Delta \boldsymbol{v}$ ). If velocity is increasing from $v_{1}$ to $v_{2}$, the acceleration is in the same direction as the velocity. (Copyright © McGraw-Hill Education)

Instantaneous acceleration is defined as the limit of the ratio of the velocity change to the elapsed time as the time interval approaches zero. When the acceleration is constant, the average acceleration and the instantaneous acceleration are equal.

If a body, moving along a straight line, is accelerated from a speed of 10 to $90 \mathrm{ft} / \mathrm{s}(3$ to $27 \mathrm{~m} / \mathrm{s})$ in 4 s , then the average change in speed per second is $(90-10) / 4=20 \mathrm{ft} / \mathrm{s}$ or $(27-3) / 4=6 \mathrm{~m} / \mathrm{s}$ in each second. This is written 20 ft per second per second or $20 \mathrm{ft} / \mathrm{s}^{2}$, or 6 m per second per second or $6 \mathrm{~m} / \mathrm{s}^{2}$. Accelerations are commonly expressed in feet per second per second, meters per second per second, or any similar units.

Whenever a body is acted upon by an unbalanced force, it will undergo acceleration. If it is moving in a constant direction, the acting force will produce a continuous change in speed. If it is moving with a constant speed, the acting force will produce an acceleration consisting of a continuous change of direction. In the general case, the acting force may produce both a change of speed and a change of direction. See also: Force.

## Angular acceleration

Angular acceleration is a vector quantity representing the rate of change of angular velocity of a body experiencing rotational motion. If, for example, at an instant $t_{1}$, a rigid body is rotating about an axis with an angular velocity $\omega_{1}$, and if, at a later time $t_{2}$, it has an angular velocity $\omega_{2}$, the average angular acceleration $\bar{\alpha}$ (which is expressed in radians per second per second) is given by equation (2):

$$
\begin{equation*}
\bar{\alpha}=\frac{\omega_{2}-\omega_{1}}{t_{2}-t_{1}}=\frac{\Delta \omega}{\Delta t} \tag{2}
\end{equation*}
$$

The instantaneous angular acceleration is given by $\alpha=d \omega / d t$.


Fig. 2 Radial acceleration $\left(a_{r}\right)$ and tangential acceleration $\left(a_{t}\right)$ in circular motion.

Consequently, if a rigid body is rotating about a fixed axis with an angular acceleration of magnitude $\alpha$ and an angular speed of $\omega_{0}$ at a given time, then the angular speed at a later time $t$ is given by equation (3):

$$
\begin{equation*}
\omega=\omega_{0}+\alpha t \tag{3}
\end{equation*}
$$

A simple calculation shows that the angular distance $\theta$ traversed in this time is expressed by equation (4):

$$
\begin{align*}
\theta=\bar{\omega} t & =\left[\frac{\omega_{0}+\left(\omega_{0}+\alpha t\right)}{2}\right] t  \tag{4}\\
& =\omega_{0} t+{ }^{1} / 2 \alpha t^{2}
\end{align*}
$$

In Fig. 2, a particle is shown moving in a circular path of radius $R$ about a fixed axis through $O$ with an angular velocity of $\omega$ radians/s and an angular acceleration of $\alpha$ radians $/ s^{2}$. This particle is subject to a linear acceleration that, at any instant, may be considered to be composed of two components: a radial component $a_{r}$ and a tangential component $a_{t}$. See also: rotational motion.

## Radial acceleration

When a body moves in a circular path with constant linear speed at each point in its path, it is also being constantly accelerated toward the center of the circle under the action of the force required to constrain it to move in its circular path. This acceleration toward the center of path is called radial acceleration. In Fig. 2, the radial acceleration, sometimes called centripetal acceleration, is shown by the vector $a_{r}$. The magnitude of its value is $v^{2} / R$, or $\omega^{2} / R$, where $v$ is the instantaneous linear velocity. This centrally directed acceleration is necessary to keep the particle moving in a circular path. See also: centripetal force.

## Tangential acceleration

The component of linear acceleration tangent to the path of a particle subject to an angular acceleration about the axis of rotation is called tangential acceleration. In Fig. 2, the tangential acceleration is shown by the vector $a_{t}$. The magnitude of its value is $\alpha R$.

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## Keywords

acceleration; angular acceleration; change of velocity; direction; instantaneous acceleration; radial acceleration; speed; tangential acceleration; time; velocity

## Test Your Understanding

1. What is acceleration and how is it measured?
2. If an object is gaining speed while spinning about its own axis, what type of acceleration is this object undergoing?
3. When a body moves in a circular path, how do the directions of its radial acceleration and tangential acceleration differ?
4. Critical Thinking: Is Earth accelerating or moving at a constant speed? Explain your answer.
5. Critical Thinking: An object that is moving in a straight line accelerates from a speed of $12 \mathrm{~m} / \mathrm{s}$ to $30 \mathrm{~m} / \mathrm{s}$ in 2 seconds. What is its acceleration?

## Additional Readings

D. Halliday, R. Resnick, and J. Walker, Fundamentals of Pbysics, 10th ed., Wiley, 2014

National Aeronautics and Space Administration, Glenn Research Center: Angular Displacement, Velocity, Acceleration

