

Conservation of momentum

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The principle that, when a system of masses is subject only to forces that masses of the system exert on one another, the total vector momentum of the system is constant. Since vector momentum is conserved, in problems involving more than one dimension the component of momentum in any direction will remain constant. The principle of conservation of momentum holds generally and is applicable in all fields of physics. In particular, momentum is conserved even if the particles of a system exert forces on one another or if the total mechanical energy is not conserved.

Use of the principle of conservation of momentum is fundamental in the solution of collision problems. *See also:* COLLISION (PHYSICS).

If a person standing on a well-lubricated cart steps forward, the cart moves backward. One can explain this result by momentum conservation, considering the system to consist of cart and human. If both person and cart are originally at rest, the momentum of the system is zero. If the person then acquires forward momentum by stepping forward, the cart must receive a backward momentum of equal magnitude in order for the system to retain a total momentum of zero.

When the principle of conservation of momentum is applied, care must be taken that the system under consideration is really isolated. For example, when a rough rock rolls down a hill, the isolated system would have to consist of the rock plus the earth, and not the rock alone, since momentum exchanges between the rock and the earth cannot be neglected.

Rocket propulsion

The propulsion of a rocket through space can be explained in terms of momentum conservation. Hot gases produced by the combustion of the fuel are expelled at high speed from the rear of the rocket. Although the total mass of these hot gases may not be large, the gases move with such a high velocity that the total momentum associated with them is appreciable. The momentum of the gases is directed backward. For momentum to be conserved, the rocket must acquire an equal momentum in the forward direction. If the rocket carries all the materials needed for the combustion of its fuel, its propulsion does not require air, and it can move through empty space. *See also:* PROPULSION; ROCKET.

Exploding bomb

An exploding bomb gives another application of the conservation of momentum. The total resultant vector momentum of all the pieces of the bomb immediately after explosion must equal the momentum of the unexploded bomb just before the explosion. *See also:* EXPLOSIVE.

Proof of principle

The principle of conservation of momentum follows directly from Newton's second and third laws. While the principle will be proved here only for the straight-line motion of a two-particle system, it can be generalized to systems containing any number of particles. A particle is a mass with dimensions so small that rotational effects are negligible. Momentum will also be conserved for rigid bodies large enough that rotation must be considered, since rigid bodies can be treated as assemblies of many particles.

For the one-dimensional motion of an isolated two-particle system, Newton's third law states that the force F_{12} that particle 1 exerts on particle 2 is equal in magnitude and opposite in direction to the force F_{21} that particle 2 exerts on particle 1. Thus Eq. (1) holds.

$$F_{21} = -F_{12} \quad (1)$$

By use of Newton's second law this equation can be expressed in terms of the momenta $m_1 v_1$ and $m_2 v_2$ of particles 1 and 2, respectively, where m_1 , m_2 , v_1 , and v_2 are the masses and velocities of particles 1 and 2, respectively. Then Eq. (2) holds.

$$m_1 \frac{dv_1}{dt} = -m_2 \frac{dv_2}{dt} \quad (2)$$

Integration gives Eq. (3),

$$m_1 v_1 + m_2 v_2 = c \quad (3)$$

where c is a constant. This equation expresses the conservation of momentum for two particles moving in the same straight line.

Finally it should be mentioned that angular and linear momentum are independent quantities. A complete description of a system must include both quantities. The angular momentum of a system is conserved under quite general conditions. *See also:* ANGULAR MOMENTUM; CONSERVATION OF ENERGY; MOMENTUM.

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Additional Readings

N. J. Giordano, *Reasoning and Relationships*, Brooks/Cole, 2010

L. D. Kirkpatrick and G. E. Francis, *Physics: A Conceptual World View*, 7th ed., Brooks/Cole, 2010

R. A. Serway and J. W. Jewett, *Principles of Physics*, 5th ed., Brooks/Cole, 2013