Gravitation

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Key Concepts

- Gravitation describes the mutual attraction between all masses and particles of matter in the universe.
- Newton's law of gravitation describes the force of attraction between two objects as an inverse square relationship. This force is directly related to the product of the masses of the objects and inversely related to the square of the distance between the objects.
- The gravitational constant, G, is a conversion factor between the units used for mass and distance and the units used for force.
- Mass and weight are not equivalent. The mass of an object is a measure of the object's inertia, which does not change with location, whereas the weight of an object is a measure of the force of gravity at that location.
- Gravity is not the same as gravitation. Gravity describes the intensity of gravitational forces on a local scale, usually on a the surface of the Earth or another celestial body, whereas gravitation refers to a universal phenomenon.
- Einstein's field theory of gravitation (general relativity) accounts for the time delay between the action of gravity experienced by an object and the motion of the source describing gravitation as a geometric property of space-time.

The mutual attraction between all masses and particles of matter in the universe. In a sense this is one of the best-known physical phenomena. During the eighteenth and nineteenth centuries gravitational astronomy, based on Newton's laws, attracted many of the leading mathematicians and was brought to such a pitch that it seemed that only extra numerical refinements would be needed in order to account in detail for the motions of all celestial bodies. In the twentieth century, however, Albert Einstein with his general theory of relativity and the concurrent development of quantum mechanics shattered this complacency (**Fig. 1**). The subject is currently in a healthy state of flux.

Until the seventeenth century, the sole recognized evidence of this phenomenon was the gravitational attraction at the surface of the Earth. Only vague speculation existed that some force emanating from the Sun kept the planets in their orbits. Such a view was expressed by Johannes Kepler, the author of the laws of planetary

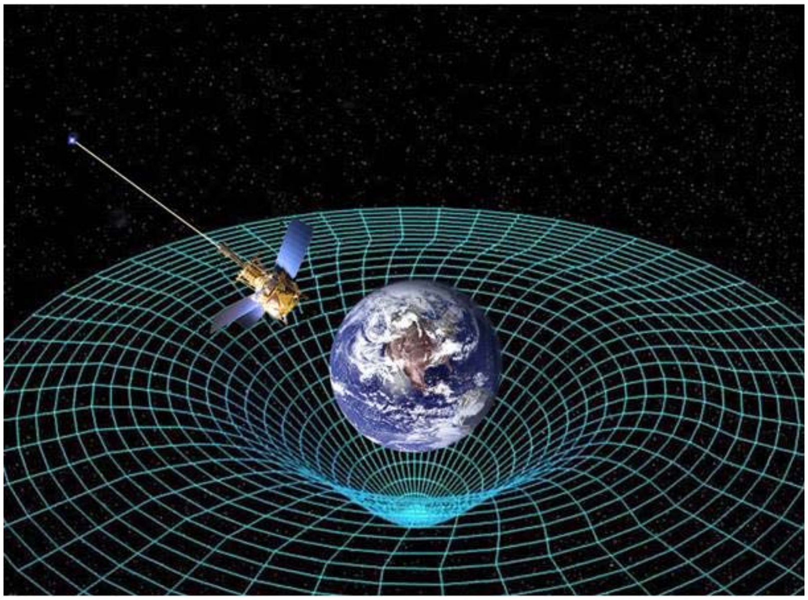

Fig. 1 NASA's Gravity Probe B mission, launched in 2004, confirmed the warping of space and time around a gravitational body, according to Albert Einstein's general theory of relativity. (Credit: NASA)

motion, to justify his empirical laws. But a proper formulation for such a force had to wait until Isaac Newton founded Newtonian mechanics, with his three laws of motion, and discovered, in calculus, the necessary mathematical tool. *See also:* CELESTIAL MECHANICS; NEWTON'S LAWS OF MOTION; PLANET.

Newton's law of gravitation

Newton's law of universal gravitation states that every two particles of matter in the universe attract each other with a force that acts in the line joining them, the intensity of which varies as the product of their masses and inversely as the square of the distance between them. Or, the gravitational force F exerted between two particles with masses m_1 and m_2 separated by a distance d is given by Eq. (1), where G is called the constant of gravitation.

$$F = \frac{Gm_1 m_2}{d^2} \tag{1}$$

A force varying with the inverse-square power of the distance from the Sun had been already suggested—notably by R. Hooke but also by other contemporaries of Newton, such as E. Halley and C. Wren—but this had been applied only to circular planetary motion. The credit for accounting for, and partially correcting, Kepler's laws and for setting gravitational astronomy on a proper mathematical basis is wholly Newton's.

Newton's theory was first published in the *Principia* in 1686. According to Newton, it was formulated in principle in 1666 when the problem of elliptic motion in the inverse-square force field was solved. But publication was delayed in part because of the difficulty of proceeding from the "particles" of the law to extended bodies such as the Earth. This difficulty was overcome when Newton established that, under his law, bodies having spherically symmetrical distribution of mass attract each other as if all their mass were concentrated at their respective centers.

Newton verified that the gravitational force between the Earth and the Moon, necessary to maintain the Moon in its orbit, and the gravitational attraction at the surface of the Earth were related by an inverse-square law of force. Let E be the mass of the Earth, assumed to be spherically symmetrical with radius R. Then the force exerted by the Earth on a small mass m near the Earth's surface is given by Eq. (2), and the acceleration of gravity on the Earth's surface, g, by Eq. (3).

$$F = \frac{GEm}{R^2} \tag{2}$$

$$g = \frac{GE}{R^2} \tag{3}$$

Let a be the mean distance of the Moon from the Earth, M the Moon's mass, and P the Moon's sidereal period of revolution around the Earth. If the motions in the Earth-Moon system are considered to be unaffected by external forces (principally those caused by the Sun's attraction), Kepler's third law applied to this system is given by Eq. (4).

$$\frac{4\pi^2 a^3}{P^2} = G(E + M) \tag{4}$$

Equations (3) and (4), on elimination of G, give Eq. (5).

$$g = 4\pi^2 \frac{E}{E + M} \frac{a^2}{R^2} \frac{a}{P^2} \tag{5}$$

Now the Moon's mean distance from the Earth is $a = 60.27R = 3.84 \times 10^8$ m (2.39×10^5 mi), and the sidereal period of revolution is $P = 27.32$ days $= 2.361 \times 10^6$ s. These data give, with $E/M = 81.35$, $g = 9.77$ m/s^2 (32.1 ft/s^2), which is close to the observed value.

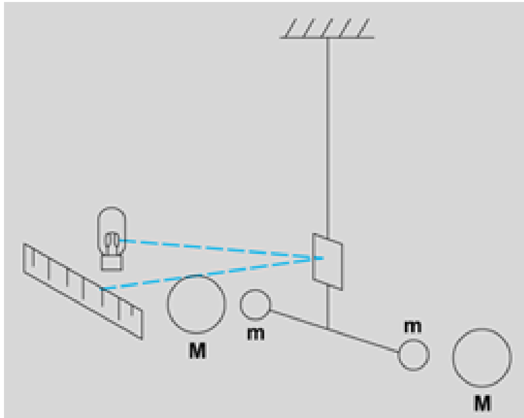

Fig. 2 Diagram of the torsion balance.

This calculation corresponds in essence to that made by Newton in 1666. At that time the ratio a/R was known to be about 60, but the Moon's distance in miles was not well known because the Earth's radius R was erroneously taken to correspond to 60 mi (97 km) per degree of latitude instead of 69 mi (111 km). As a consequence, the first test was unsatisfactory. But the discordance was removed in 1671 when the measurement of an arc of meridian in France provided a reliable value for the Earth's radius.

Gravitational constant

Equation (3) shows that the measurement of the acceleration due to gravity at the surface of the Earth is equivalent to finding the product G and the mass of the Earth. Determining the gravitational constant by a suitable experiment is therefore equivalent to "weighing the Earth."

In 1774, G was determined by measuring the deflection of the vertical by the attraction of a mountain. This method is much inferior to the laboratory method in which the gravitational force between known masses is measured. In the torsion balance two small spheres, each of mass m, are connected by a light rod, suspended in the middle by a thin wire. The deflection caused by bringing two large spheres each of mass M near the small ones on opposite sides of the rod is measured, and the force is evaluated by observing the period of oscillation of the rod under the influence of the torsion of the wire (see **Fig. 2**). This is known as the Cavendish experiment, in honor of H. Cavendish, who achieved the first reliable results by this method in 1797–1798. More recent determinations using various refinements yield the results: constant of gravitation $G = 6.67 \times 10^{-11}$ SI (mks) units; mass of Earth $= 5.98 \times 10^{24}$ kg. The result of the best available laboratory measurements, announced in 2000, is $G = (6.674215 \pm 0.000092) \times 10^{-11}$ in SI (mks) units. As a result, the mass of the Earth is $(5.972254 \pm 0.000082) \times 10^{24}$ kg and the Sun's mass is $(1.988435 \pm 0.000027) \times 10^{30}$ kg.

In a more basic sense, G is a conversion factor between the units used for mass and distance and the units used for force. This definition must give the same force unit as the electromagnetic methods. These other methods are much more accurate than our current knowledge of *G*. Hopefully, improvements in the value of *G* will give the same size for the unit of force. Thus, knowing *G* more accurately will check the consistency of our definition of force.

In Newtonian gravitation, *G* is an absolute constant, independent of time, place, and the chemical composition of the masses involved. Partial confirmation of this was provided before Newton's time by the experiment attributed to Galileo in which different weights released simultaneously from the top of the Tower of Pisa reached the ground at the same time. Newton found further confirmation, experimenting with pendulums made out of different materials. Early in this century, R. Eötvös found that different materials fall with the same acceleration to within 1 part in 10^7. The accuracy of this figure has been extended to 1 part in 10^{11}, using aluminum and gold, and to 0.9×10^{-12} with a confidence of 95%, using aluminum and platinum.

With the discovery of antimatter, there was speculation that matter and antimatter would exert a mutual gravitational repulsion. But experimental results indicate that they attract one another according to the same laws as apply to matter of the same kind. *See also:* ANTIMATTER.

A cosmology with changing physical "constants" was first proposed in 1937 by P. A. M. Dirac. Field theories applying this principle have since been proposed by P. Jordan and D. W. Sciama and, in 1961, by C. Brans and R. H. Dicke. In these theories *G* is diminishing; for instance, Brans and Dicke suggest a change of about 2×10^{-11} per year. This would have profound effects on phenomena ranging from the evolution of the universe to the evolution of the Earth. There is no firm evidence at present to support a time variation of *G*. For instance, detailed analyses of the motion of the Moon using the lunar laser ranging data, the analysis of solar system data, and especially the ranging data using Viking landers on Mars, and the timing analysis of the binary pulsar PSR B1913 + 16 have all resulted in only upper limits on a possible temporal variation of *G*. The present upper limit on the relative rate of time variation of *G* is a few parts in 10^{14} per year. This result is theory-independent; that is, it is based purely on observations without relying on any particular theory in which *G* is presumed to vary with time. *See also:* PULSAR.

Mass and weight

In the equations of motion of Newtonian mechanics, the mass of a body appears as inertial mass, a measure of resistance to acceleration, and as gravitational mass in the expression of the gravitational force. The equality of these masses is confirmed by the Eötvös experiment. It justifies the assumption that the motion of a particle in a gravitational field does not depend on its physical composition. In Newton's theory the equality can be said to be a coincidence, but not in Einstein's theory, where this equivalence becomes a cornerstone of relativistic gravitation.

While mass in Newtonian mechanics is an intrinsic property of a body, its weight depends on certain forces acting on it. For example, the weight of a body on the Earth depends on the gravitational attraction of the Earth on the body and also on the centrifugal forces due to the Earth's rotation. The body would have lower weight on the Moon, even though its mass would remain the same. *See also:* CENTRIFUGAL FORCE.

Gravity

This should not be confused with the term gravitation. Gravity is the older term, meaning the quality of having weight, and so came to be applied to the tendency of downward motion on the Earth. Gravity or the force of gravity is today used to describe the intensity of gravitational forces, usually on the surface of the Earth or another celestial body. So gravitation refers to a universal phenomenon, while gravity refers to its local manifestation.

A rotating planet is oblate (or flattened at the poles) to a degree depending on the ratio of the centrifugal to the gravitational forces on its surface and on the distribution of mass in its interior. The variation of gravity on the surface of the Earth depends on these factors and is further complicated by irregular features such as oceans, continents, and mountains. It is investigated by gravity surveys and also through the analysis of the motion of artificial satellites. Because of the irregularities, no mathematical formula has been found that satisfactorily represents the gravitational field of the Earth, even though formulas involving hundreds of terms are used. The problem of representing the gravitational field of the Moon is even harder because the surface irregularities are proportionately much larger. *See also:* EARTH'S GRAVITY FIELD.

In describing gravity on the surface of the Earth, a smoothed-out theoretical model is used, to which are added gravity anomalies, produced in the main by the surface irregularities.

Gravity waves are waves in the oceans or atmosphere of the Earth whose motion is dynamically governed by the Earth's gravitational field. They should not be confused with gravitational waves, which are discussed below.

Gravitational potential energy

This describes the energy that a body has by virtue of its position in a gravitational field. If two particles with masses m_1 and m_2 are a distance r apart and if this distance is slightly increased to $r + \Delta r$, then the work done against the gravitational attraction is $Gm_1\ m_2\Delta r/r^2$. If the distance is increased by a finite amount, say from r_1 to r_2, the work done is given by Eq. (6). If $r_2 \to \infty$, Eq. (7)

$$W_{r_1,r_2} = Gm_1 m_2 \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= Gm_1 m_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \tag{6}$$

$$W_{r_1,\infty} = \frac{Gm_1m_2}{r_1} \tag{7}$$

holds.

If one particle is kept fixed and the other brought to a distance *r* from a very great distance (infinity), then the work done is given by Eq. (8).

$$-U = \frac{-Gm_1m_2}{r} \tag{8}$$

This is called the gravitational potential energy; it is (arbitrarily) put to zero for infinite separation between the particles. Similarly, for a system of *n* particles with masses $m_1, m_2, \ldots, m_n$ and mutual distance r_{ij} between m_i and m_j, the gravitational potential energy $-U$ is the work done to assemble the system from infinite separation (or the negative of the work done to bring about an infinite separation), as shown in Eq. (9).

$$-U = -G\sum_{i<j}\frac{m_im_j}{r_{ij}} \tag{9}$$

A closely related quantity is gravitational potential. The gravitational potential of a particle of mass *m* is given by Eq. (10),

$$V = \frac{-Gm}{r} \tag{10}$$

where *r* is distance measured from the mass. The gravitational force exerted on another mass *M* is *M* times the gradient of *V*. If the first body is extended or irregular, the formula for *V* may be extremely complicated, but the latter relation still applies. *See also:* POTENTIALS.

A good illustration of gravitational potential energy occurs in the motion of an artificial satellite in a nearly circular orbit around the Earth which is affected by atmospheric drag. Because of the frictional drag the total energy of the satellite in its orbit is reduced, but the satellite actually moves faster. The explanation for this is that it moves closer to the Earth and loses more in gravitational potential energy than it gains in kinetic energy. *See also:* ENERGY.

Similarly, in its early evolution a star contracts, with the gravitational potential energy being transformed partly into radiation, so that it shines, and partly into kinetic energy of the atoms, so that the star heats up until it is hot enough for thermonuclear reactions to start. *See also:* STELLAR EVOLUTION.

Another related phenomenon is that of speed of escape. A projectile launched from the surface of the Earth with speed less than the speed of escape will return to the surface of the Earth; but it will not return if its initial speed is greater (atmospheric drag is neglected). For a spherical body with mass M and radius R, the speed of escape from its surface is given by Eq. (11).

$$V_e = \left(\frac{2MG}{R}\right)^{1/2} \tag{11}$$

For the Earth, V_e is 11.2 km/s (7.0 mi/s); for the Moon, it is 2.4 km/s (1.5 mi/s), which explains why the Moon cannot retain an atmosphere such as the Earth's. By analogy, a black hole can be considered a body for which the speed of escape from the surface is greater than the speed of light, so that light cannot escape; however, the analogy is not really exact since Newtonian mechanics is not valid on the scale of the universe. *See also:* BLACK HOLE; ESCAPE VELOCITY.

Application of Newton's law

In modern times, Eq. (5)—in a modified form with appropriate refinements to allow for the Earth's oblateness and for external forces acting on the Earth-Moon system—has been used to compute the distance to the Moon. The results have been superseded in accuracy only by radar measurements and observations of corner reflectors placed on the lunar surface.

Newton's theory passed a much more stringent test than the one described above when he was able to account for the principal departures from Kepler's laws in the motion of the Moon. Such departures are called perturbations. A notable triumph of the theory occurred when the observed perturbations in the motion of the planet Uranus enabled J. C. Adams in 1845 and U. J. Leverrier in 1846 independently to predict the existence and calculate the position of a hitherto-unobserved planet, later called Neptune. When yet another planet, Pluto, was discovered in 1930, its position and orbit were strikingly similar to predictions based on the method used to discover Neptune. But the discovery of Pluto must be ascribed to the perseverance of the observing astronomers; it is not massive enough to have revealed itself through the perturbations of Uranus and Neptune. *See also:* PERTURBATION (ASTRONOMY).

F. W. Bessel observed nonuniform proper motions of Sirius and Procyon and inferred that each was gravitationally deflected by an unseen companion. It was only after his death that these bodies were telescopically observed, and they both later proved to be white dwarfs. More recently evidence has been accumulated for the existence of some planetary masses around stars. The discovery of black holes (which will never be directly observed) hinges in part on a visible star showing evidence for having a companion of sufficiently high mass (so that its gravitational collapse can never be arrested) and on observations of the motions of stars near galactic centers. *See also:* BINARY STAR.

Newton's theory supplies the link between the observed motion of celestial bodies and certain physical properties, such as mass and sometimes shape. Knowledge of stellar masses depends basically on the application of the theory to binary-star systems. Analysis of the motions of artificial satellites placed in orbit around the Earth has revealed refined information about the gravitational field of the Earth and of the Earth's atmosphere. Similarly, satellites placed in orbit around the Moon have yielded information about its gravitational field, and other space vehicles have yielded the best information to date on the masses and gravitational fields of other planets. *See also:* SATELLITE (SPACECRAFT); SPACE PROBE.

Newtonian gravitation has been applied to the motion of stars in galaxies as well as the motion of galaxies in clusters of galaxies. Careful observations of the motions of stars and gas (using the 21-cm line of neutral hydrogen) in the outer reaches of spiral galaxies have shown that the speed of rotation far from the center of a galaxy does not diminish with distance as would be expected on the basis of the gravitational influence of the rest of the stars as implied by the light distribution in the galaxy, but stays essentially constant. This circumstance leads to a rotation curve (the plot of the rotational speed versus the radius away from the center of a disk-shaped spiral galaxy) that is flat at large distances from the nucleus of the galaxy. The phenomenon associated with flat rotation curves for the outer regions of spiral galaxies has been well established, and is now generally interpreted in terms of the existence of the so-called dark matter, that is, excess matter that is invisible. Of course, the discrepancy with the prediction of Newtonian gravitation could be due to the breakdown of the theory. Newtonian gravitation and its relativistic generalization (general relativity) have been well tested in the solar system, but significant deviations over larger scales cannot be ruled out in principle. Though phenomenological modifications of the theory have been proposed to explain the discrepancy, no alternative theory that is based on fundamental physical principles exists. Therefore, the discrepancy is generally attributed to the existence of dark matter halos in which the visible galaxies are presumed to be embedded. Dark halo models together with rotation curves are used to calculate the density of dark matter in galaxies. It is estimated that the visible mass of a galaxy is only about 10% of its total mass. Furthermore, the discrepancy between the luminous and dynamical masses is thought to be larger for clusters of galaxies. Even more dark matter is needed if certain models of cosmology, such as the inflationary scenario, are taken into account. The result is that more than 90% of the mass of the universe could be in the form of dark matter. Proposals for the nature of dark matter range from exotic subatomic particles to black holes. The determination of the nature of dark matter constitutes a fundamental problem in astrophysics and cosmology. *See also:* COSMOLOGY; DARK MATTER; GALAXY, EXTERNAL; INFLATIONARY UNIVERSE COSMOLOGY; MILKY WAY GALAXY; UNIVERSE.

Accuracy of Newtonian gravitation

Newton was the first to doubt the accuracy of his law when he was unable to account fully for the motion of the perigee in the motion of the Moon. In this case he eventually found that the discrepancy was largely removed if the solution of the equations was more accurately developed. Further difficulties to do with the motion of the Moon were noted in the nineteenth century, but these were eventually resolved when it was found that there were appreciable fluctuations in the rate of rotation of the Earth, so that it was the system of timekeeping and not the gravitational theory that was at fault. *See also:* EARTH ROTATION AND ORBITAL MOTION.

A more serious discrepancy was discovered by Leverrier in the orbit of Mercury. Because of the action of the other planets, the perihelion of Mercury's orbit advances. But allowance for all known gravitational effects still left an observed motion of about 43 seconds of arc per century unaccounted for by Newton's theory. Attempts to account for this by adding an unknown planet or by drag with an interplanetary medium were unsatisfactory, and a very small change was suggested in the exponent of the inverse square of force. This particular discordance was accounted for by Einstein's general theory of relativity in 1916.

Newtonian mechanics considered gravity as a direct action-at-a-distance and in the early days did not properly account for the time delay between the action of gravity experienced by a body and the motion of the source. General relativity, on the other hand, considers gravitational effects as being geometric and time-related. *See also:* RELATIVITY.

Gravitational lens

From a relativistic point of view, light is expected to be deflected when it passes through a gravitational field. An analogy can be made to the refraction of light passing through a lens. For example, a galaxy situated between an observer and a more distant source will have a focusing effect, and this accounts for some of the observed properties of quasistellar objects. The multiple images of the quasar (Q0957+561 A,B) are almost certainly caused by the light from a single body passing through a gravitational lens. While this is the best-studied gravitational lens, many other examples of this phenomenon have been discovered. *See also:* GRAVITATIONAL LENS; QUASAR.

Testing of gravitational theories

One of the greatest difficulties in investigating gravitational theories is the weakness of the gravitational coupling of matter. For instance, the gravitational interaction between a proton and an electron is weaker by a factor of about 5×10^{-40} than the electrostatic interaction. (If gravitation alone bound the hydrogen atom, then the radius of the first Bohr orbit would be 10^{13} light-years, or about 1000 times the Hubble radius!) Of the four basic interactions (that is, strong, electromagnetic, weak, and gravitational) that are known at present, gravitation is by far the weakest force in nature. [A unification of the electromagnetic and weak interactions into the so-called electroweak interaction has been carried out, and further unification of this interaction with the strong (nuclear) force exists. So far, no successful realistic unification with gravitational interactions has been accepted.] *See also:* ELECTROWEAK INTERACTION; FUNDAMENTAL INTERACTIONS.

Many attempts have been made to discover Yukawa-type forces that would couple to matter with strengths close to that of gravity. These forces are expected to have finite ranges, however, in contrast to the infinite range of universal gravitation. Thus, deviations from Newton's inverse-square law of gravitation could reveal the presence of such interactions. These forces also appear in certain theories that attempt to unify gravity with the other fundamental interactions. Compelling evidence for the existence of any anomalous macroscopic force has not been found.

Results of experiments to search for new spin-dependent interactions place strict upper limits on anomalous spin-dependent couplings. Furthermore, torsion-balance experiments using ordinary matter have been interpreted as indicating the universality of free fall for matter as well as antimatter.

There is no firm evidence at present for any deviation from the Newtonian law of gravitation in the nonrelativistic regime.

Relativistic theories

Before Newton, detailed descriptions were available of the motions of celestial bodies—not just Kepler's laws but also empirical formulas capable of representing with fair accuracy, for their times, the motion of the Moon. Newton replaced description by theory, but in spite of his success and the absence of a reasonable alternative, the theory was heavily criticized, not least with regard to its requirement of "action at a distance" (that is, through a vacuum). Newton himself considered this to be "an absurdity," and he recognized the weaknesses in postulating in his system of mechanics the existence of preferred reference systems (that is, inertial reference systems) and an absolute time. Newton's theory is a superb mathematical one that represents the observed phenomena with remarkable accuracy.

The investigation of electric and magnetic phenomena culminated in the second half of the nineteenth century in the complete formulation of the laws of electromagnetism by J. C. Maxwell. Maxwell based his theory on M. Faraday's field concept. The electromagnetic field propagates with the speed of light; in fact, Maxwell's theory unified the science of optics with electricity and magnetism. Maxwell's theory of the electromagnetic field was extended and strengthened with the subsequent observations of electromagnetic waves by H. Hertz and the successes of the theory of electrons developed by H. A. Lorentz. *See also:* ELECTROMAGNETIC RADIATION; MAXWELL'S EQUATIONS.

The theory of relativity grew from attempts to describe electromagnetic phenomena in moving systems. No physical effect can propagate with a speed exceeding that of light in vacuum; therefore, Newton's theory must be the limiting case of a field theory in which the speed of propagation approaches infinity. Einstein's field theory of gravitation (general relativity) is based on the identification of the gravitational field with the curvature of space-time (**Fig. 1**). The geometry of space-time is affected by the presence of matter and radiation. The relationship between mass-energy and the space-time curvature is therefore a relativistic generalization of the Newtonian law of gravitation. The relativistic theory is mathematically far more complicated than Newton's. Instead of the single Newtonian potential described above, Einstein worked with 10 quantities that form a tensor. *See also:* TENSOR ANALYSIS.

Principle of equivalence

An important step in Einstein's reasoning is his principle of equivalence, saying that a uniformly accelerated reference system imitates completely the behavior of a uniform gravitational field. Imagine, for instance, a scientist in a space capsule infinitely far out in empty space so that the gravitational force on the capsule is negligible. Everything would be weightless; bodies would not fall; and a pendulum clock would not work. But now imagine the capsule to be accelerated by some agency at the uniform rate of 981 cm/s^2 (32.2 ft/s). Everything in the capsule would then behave as if the capsule were stationary on its launching pad on the surface of the Earth and therefore subject to the Earth's gravitational field. But after its original launching, when the capsule is in free flight under the action of gravitational forces exerted by the various bodies in the solar system, its contents will behave as if it were in the complete isolation suggested above. This principle requires that all bodies fall in a gravitational field with precisely the same acceleration, a result that is confirmed by the Eötvös experiment mentioned earlier. Also, if matter and antimatter were to repel one another, it would be a violation of the principle. *See also:* FREE FALL.

Einstein's theory requires that experiments should have the same results irrespective of the location or time. This has been said to amount to the "strong" principle of equivalence.

Classical tests

The ordinary differential equations of motion of Newtonian gravitation are replaced in general relativity by a nonlinear system of partial differential equations for which general solutions are not known. Apart from a few special cases, knowledge of solutions comes from methods of approximation. For instance, in the solar system, speeds are low so that the quantity υ/c (υ is the orbital speed and c is the speed of light) will be small (about 10^{-4} for the Earth). The equations and solutions are expanded in powers of this quantity; for instance, the relativistic correction for the motion of the perihelion of Mercury's orbit is adequately found by considering no terms smaller than $(\upsilon/c)^2$. This is called the post-Newtonian approximation. (Another approach is the weak-field approximation.)

Einstein's theory has appeared to pass three famous tests. First, it accounted for the full motion of the perihelion of the orbit of Mercury. (Mercury is the most suitable planet, because it is the fastest-moving of the major planets and has a high eccentricity, so that its perihelion is relatively easily studied.) Second, the prediction that light passing a massive body would be deflected has been confirmed with an accuracy of about 5%. Third, Einstein's theory predicted that clocks would run more slowly in strong gravitational fields compared to weak ones; interpreting atoms as clocks, spectral lines would be shifted to the red in a gravitational field. This, again, has been confirmed with moderate accuracy. Locations on the Earth's surface can be determined with great accuracy using the artificial satellites that make up the Global Positioning System (GPS). The clocks in that system must make relativistic corrections for both their motions relative to the Earth (special theory) and their gravitational potential relative to the Earth's surface (general theory). *See also:* SATELLITE NAVIGATION SYSTEMS.

Predictions of the theory have been confirmed in an experiment in which radar waves were bounced off Mercury; the theory predicts a delay of about 2×10^{-4} s in the arrival time of a radar echo when Mercury is on the far side of the Sun and close to the solar limb. Tests, similar in principle, have been conducted using observations of the Mariner space vehicles, the accuracy of confirmation being in the region of 4%. A greater level of accuracy has been achieved, by better than an order of magnitude, using data from transponders on Viking orbiters and landers on Mars. Furthermore, the deflection of microwave radiation passing close to the Sun has been observed using radio interferometry with a baseline of 22 mi (35 km). The amount of bending that has been found is 1.015 ± 0.011 times the amount predicted by general relativity. In another test, the precession of a gyroscope in orbit around the Earth is to be studied for evidence of the so-called geodetic precession as well as the precession due to the gravitomagnetic field of the rotating Earth. The lunar laser-ranging data have been used to measure the de Sitter precession of the Moon's orbit. This is due to the geodetic precession of the Earth-Moon orbital angular momentum in the gravitational field of the Sun. It amounts to an advance in the lunar node and perigee by about 2 seconds of arc per century, a prediction first made by W. de Sitter in 1916 soon after the advent of general relativity. Such small secular effects are suitable for study since they accumulate in time. Other periodic (noncumulative) orbital effects have until recently been too small to observe. But the current revolution in observational techniques and accuracy has changed the situation; post-newtonian terms are now routinely included in many calculations of the orbits of planets and space vehicles, and comparison with observations will furnish tests of the theory.

Mach's principle

One of the most penetrating critiques of mechanics is due to E. Mach, toward the end of the nineteenth century. Some of his ideas can be traced back to Bishop G. Berkeley early in the eighteenth century. Out of Mach's work there has arisen Mach's principle; this is philosophical in nature and cannot be stated in precise terms. The idea is that the motion of a particle is meaningful only when referred to the rest of the matter in the universe. Geometrical and inertial properties are meaningless for an empty space, and the motion of a particle in such space is devoid of physical significance. Thus the behavior of a test particle should be determined by the total matter distribution in the universe and should not appear as an intrinsic property of an absolute space. Mach's principle suggests that gravitation and inertia are equivalent. This idea strongly influenced the development of general relativity; however, general relativity, having an absolute concept of rotation, does not fully satisfy Mach's principle.

Brans-Dicke theory

This is a classical field theory of gravitation that was developed in 1961 by Brans and Dicke on the basis of an interpretation of Mach's principle. In this theory the gravitational field is described by a tensor and a scalar, the equations of motion being the same as those in general relativity. The addition of a scalar field leads to the appearance of an arbitrary constant, whose value is not known exactly. The Brans-Dicke theory predicts that the relativistic motion of the perihelion of Mercury's orbit is reduced compared with Einstein's value, and also that the light deflection should be less. With regard to the orbit of Mercury, Dicke pointed out that if the Sun were oblate, this might account for some of the motion of the perihelion. In 1967 he announced that measurements showed a solar oblateness of about 5 parts in 100,000 (or a difference in the polar and equatorial radii of about 21 mi or 34 km). His observations and discussion are still subject to some controversy. The difference between the theory and that of general relativity can be parametrized by the number ω, where $\gamma = (1 + \omega)/(2 + \omega)$; for general relativity, $\gamma = 1$. Dicke has proposed $\omega \approx 7.5$; but the results of measurement of deflection of radiation by the Sun indicate a value of ω greater than 23, for which the predictions of the two theories would not be greatly different. Subsequent data from the Viking spacecraft imply that this constant must be greater than about 500, thus rendering the Brans-Dicke theory almost indistinguishable from general relativity.

There are, of course, many other theories not mentioned here.

Supergravity

This is the term applied to highly mathematical theories of gravitation attempting to form a part of a unified field theory in which all types of forces are included. String, loop, and M-brane theories are under investigation. *See also:* FUNDAMENTAL INTERACTIONS; QUANTUM GRAVITATION; SUPERGRAVITY; SUPERSTRING THEORY.

Gravitational waves

The existence of gravitational waves, or gravitational "radiation," was predicted by Einstein shortly after he formulated his general theory of relativity. They are now a feature of any relativity theory. Gravitational waves are "ripples in the curvature of space-time." In other words, they are propagating gravitational fields, or propagating patterns of strain, traveling at the speed of light. They carry energy and can exert forces on matter in their path, producing, for instance, very small vibrations in elastic bodies. The gravitational wave is produced by change in the distribution of some matter. It is not produced by a rotating sphere, but would result from a rotating body, or pairs of bodies, not having symmetry about their axis of rotation or from pulsars and supernova explosions. In spite of the relatively weak interaction between gravitational radiation and matter, the measurement of this radiation is now technically possible. *See also:* SUPERNOVA.

There are currently (2006) five experiments attempting to detect gravitational radiation by interferometric methods: the two LIGO (Laser Interferometer Gravitational-Wave Observatory) detectors located in the United

States (near Livingston, Louisiana, and at the Washington Hanford Nuclear Research Area), VIRGO (France and Italy), GEO 600 (Germany), and TAMA (Japan). These instruments can detect a shift of the order of 1 part in 10^{21}. It is possible, if these instruments meet their design standards, that they may be sensitive enough to detect nearby wave sources. LIGO 2, which is under design, should increase sensitivity by a factor of 10. *See also:* LIGO (LASER INTERFEROMETER GRAVITATIONAL-WAVE OBSERVATORY).

The National Aeronautics and Space Administration (NASA) and the European Space Agency (ESA) are planning a space-based interferometer, the Laser Interferometer Space Antenna (LISA). It will consist of three spacecraft in an equilateral triangle formation with arms 5×10^6 km (3×10^6 mi) long. It will be located in the Earth's orbital path about 20° behind the Earth. Communication lasers will be located in each spacecraft. This design eliminates Earth noise and avoids the problems of folded light paths. *See also:* GRAVITATIONAL RADIATION.

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Keywords

gravitation; gravity; Newton's law; mass; force of gravity; gravitational potential energy; gravitational lens; general relativity

Test Your Understanding

1. What happens to the gravitational potential energy of an object as it falls to Earth?
2. How do modern astronomers use gravitational lensing in their research?
3. Critical Thinking: Does the location of an object on Earth affect its mass and weight? Why or why not?
4. Critical Thinking: If the masses of two objects and the distance between them are all doubled, what is the effect on the force between them? Explain your answer.

Additional Readings

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