Magnetism

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The branch of science that describes the effects of the interactions between charges due to their motion and spin. These may appear in various forms, including electric currents and permanent magnets. The interactions are described in terms of the magnetic field, although the field hypothesis cannot be tested independently of the electrokinetic effects by which it is defined. The magnetic field complements the concept of the electrostatic field used to describe the potential energy between charges due to their relative positions. Special relativity theory relates the two, showing that magnetism is a relativistic modification of the electrostatic forces. The two together form the electromagnetic interactions which are propagated as electromagnetic waves, including light. They control the structure of materials at distances between the long-range gravitational actions and the short-range "strong" and "weak" forces most evident within the atomic nucleus. *See also:* ELECTROMAGNETIC RADIATION; RELATIVITY.

The term "magnetism" originates in the material magnetite, an iron ore, which produces weak natural magnets in the form of lodestones, exerting forces on each other and on pieces of iron. Peregrinus showed in 1269 that the behavior can be described in terms of magnetic poles on opposite end surfaces. The analogy between magnetic poles and electric charge greatly influenced the later development of the subject. The Earth provides an example of the subsequent explanation of magnetic behavior in terms of the flow of current and movement of charge, including the quantum state described as spin. This leaves open the question of whether or not isolated magnetic poles, or monopoles, exist as separate physical entities. *See also:* MAGNETIC MONOPOLES; MAGNETITE.

The magnetic field can be visualized as a set of lines (**Fig. 1**) illustrated by iron filings scattered on a suitable surface. The intensity of the field is indicated by the line spacing, and the direction by arrows pointing along the lines. The sign convention is chosen so that the Earth's magnetic field is directed from the north magnetic pole toward the south magnetic pole. The field can be defined and measured in various ways, including the forces on the equivalent magnetic poles, and on currents or moving charges. Bringing a coil of wire into the field, or removing it, induces an electromotive force (emf) which depends on the rate at which the number of field lines, referred to as lines of magnetic flux, linking the coil changes in time. This provides a definition of flux, Φ, in terms of the emf, e, given by Eq. (1)

$$e = -N\, d\Phi/dt \qquad \text{volts} \tag{1}$$

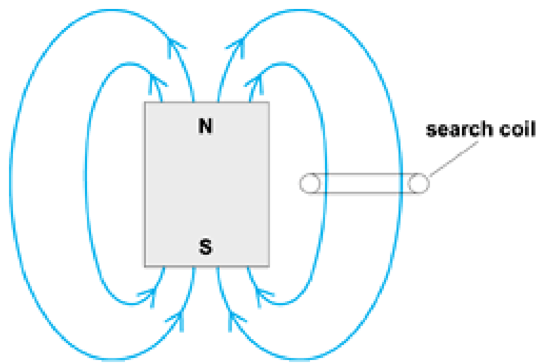

Fig. 1 **Magnetic lines of a bar magnet.**

for a coil of N turns wound sufficiently closely to make the number of lines linking each the same. The International System (SI) unit of Φ, the weber (Wb), is defined accordingly as the volt-second. The symbol B is used to denote the flux, or line, density, as in Eq. (2),

$$B = \Phi/\text{area} \tag{2}$$

when the area of the coil is sufficiently small to sample conditions at a point, and the coil is oriented so that the induced emf is a maximum. The SI unit of B, the tesla (T), is the Wb/m^2. The sign of the emf, e, is measured positively in the direction of a right-hand screw pointing in the direction of the flux lines. It is often convenient, particularly when calculating induced emfs, to describe the field in terms of a magnetic vector potential function instead of flux.

Magnetic circuits

The magnetic circuit provides a useful method of analyzing devices with ferromagnetic parts, and introduces various quantities used in magnetism. It describes the use of ferromagnetic materials to control the flux paths in a manner analogous to the role of conductors in carrying currents around electrical circuits. For example, pieces of iron may be used to guide the flux which is produced by a magnet along a path which includes an air gap (**Fig. 2**), giving an increase in the flux density, B, if the cross-sectional area of the gap is less than that of the magnet. *See also:* MAGNET; MAGNETIC MATERIALS.

The magnet may be replaced by a coil of N turns carrying a current, i, wound over a piece of iron, or ferromagnetic material, in the form of a ring of uniform cross section (**Fig. 3**). The flux linking each turn of the coil, and each turn of a secondary coil wound separately from the first, is then approximately the same, giving the same induced emf per turn [according to Eq. (1)] when the supply current, i, and hence the flux, Φ, changes in time. The arrangement is typical of many different devices. It provides, for example, an electrical transformer

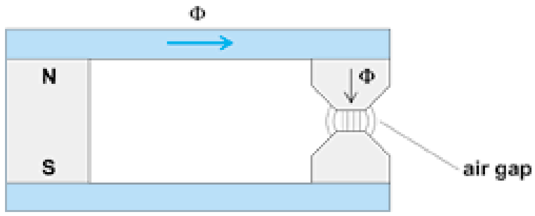

Fig. 2 **Magnetic circuit with an air gap.**

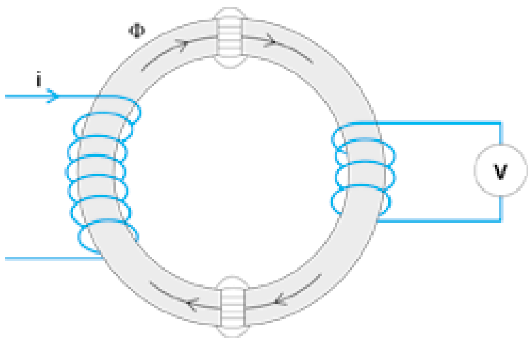

Fig. 3 **Coils coupled by a ring of ferromagnetic material of uniform cross section.**

whose input and output voltages are directly proportional to the numbers of turns in the windings. Emf's also appear within the iron, and tend to produce circulating currents and losses. These are commonly reduced by dividing the material into thin laminations. *See also:* EDDY CURRENT; TRANSFORMER.

The amount of flux produced by a given supply current is reduced by the presence of any air gaps which may be introduced to contribute constructional convenience or to allow a part to move. The effects of the gaps, and of different magnetic materials, can be predicted by utilizing the analogy between flux, Φ, and the flow of electric current through a circuit consisting of resistors connected in series (**Fig. 4**). Since Φ depends on the product, iN, of the winding current and number of turns, as in Eq. (3),

$$iN = \Phi\Re \qquad (3)$$

the ratio between them, termed the reluctance, , is the analog of electrical resistance. It may be constant, or may vary with Φ. The quantity iN is the magnetomotive force (mmf), analogous to voltage or emf in the equivalent electrical circuit. The relationship between the two exchanges the potental and flow quantities, since the magnetic mmf depends on current, i, and the electrical emf on $d\Phi/dt$. Electric and magnetic equivalent circuits are referred to as duals. *See also:* RELUCTANCE.

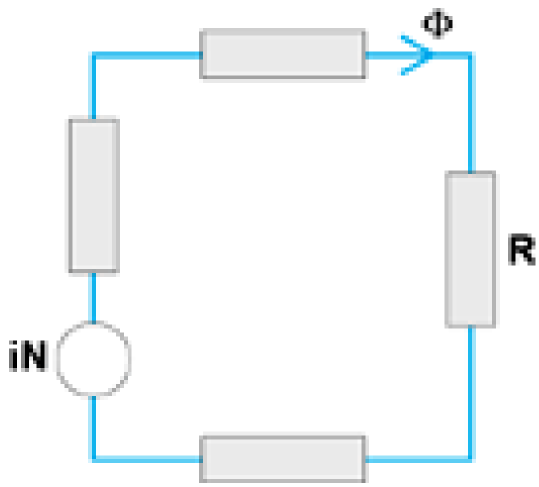

Fig. 4 **Circuit analogy. Components of a magnetic circuit carrying a flux Φ, analogous to current *i*. The reluctances of the components are analogous to resistance, *R*.**

Any part of the magnetic circuit of length *l*, in which the cross section, a, and flux density, *B*, are uniform has a reluctance given by Eq. (4). This equation parallels Eq. (5)

$$\Re = l/(a\mu) \tag{4}$$

$$R = l/(a\sigma) \tag{5}$$

for the resistance, *R*, of a conduct or of the same dimensions. The permeability, μ, is the magnetic equivalent of the conductivity, σ, of the conducting material. Using a magnet as a flux source (Fig. 2) gives an mmf which varies with the air gap reluctance. In the absence of any magnetizable materials, as in the air gaps, the permeability is given by Eq. (6)

$$\mu = \mu_0 = 4\pi \times 10^{-7} \tag{6}$$

in SI units (Wb/A-m). The quantity μ_0 is sometimes referred to as the permeability of free space. Material properties are described by the relative permeability, μ_r in accordance with Eq. (7),

$$\mu = \mu_r \mu_0 \tag{7}$$

The materials which are important in magnetic circuits are the ferromagnetics and ferrites characterized by large value of μ_r, sometimes in excess of 10,000 at low flux densities.

The various reluctances, $_1$, $_2$, . . . , forming the different parts of the magnetic circuit can be added, yielding Eq. (8),

$$\Phi = iN/(\Re_1 + \Re_2 + \Re_3 + \cdots) \quad (8)$$

when each part carries the same flux, Φ. The largest reluctances are often those of the air gaps. In iron the nonlinear relationship between mmf and flux may produce large variations in , and it is then convenient to specify *B*, and use the magnetization curve, relating *B* to the mmf per meter, to obtain the mmf, *iN*. Fringe fields in air may cause significant differences in fluxes in the different parts of the circuit, referred to as flux leakage, and these usually require corresponding adjustments to *B*. Greater accuracy is obtained by using numerical methods to compute the field.

The analogy between magnetic and electric circuits fails in that energy is dissipated in resistance but not in reluctance. Equations (1) and (3) show that a coil of *N* turns magnetizing a magnetic circuit of reluctance absorbs an energy given by Eq. (9) in time *dt*,

$$ei\,dt = iN\,d\Phi = \Re\Phi\,d\Phi \qquad \text{joules} \quad (9)$$

where *d*Φ is the flux change [Eq. (1)]. Summing shows that, if is constant, then the total energy required to increase the flux from zero to Φ is given by Eq. (10),

$$\int ei\,dt = \Re\Phi^2/2 \qquad \text{joules} \quad (10)$$

and this energy, measured in joules, is stored, not dissipated, since it is recovered when the flux is driven back to zero.

The storage of energy is a key property of a magnetic circuit or, more generally, of a magnetic field. Since the flux $\delta\Phi$ in a volume element of cross-sectional area δa is $B\ \delta a$, and the reluctance δ is proportional to the length δl [Eq. (4)], the energy required to magnetize the element is proportional to its volume. Substituting in Eq. (10) shows that the energy per unit volume is given by Eq. (11).

$$\mathfrak{E} = B^2/2\mu \qquad \text{J/m}^3 \quad (11)$$

The high permeability, μ, of the iron causes a high concentration of energy, G, in the air gaps of a magnetic circuit in which B is uniform. This is recovered if the iron surfaces are allowed to come together, while B is kept constant. Hence, the force per unit area is given by Eq. (12)

$$f = B^2/2\mu_0 \tag{12}$$

when the flux density, B, in the air is in the direction normal to the surface. The force of attraction between the two halves of the magnetic circuit in Fig. 3 is obtained by summing f over the areas of both air gaps. The attraction is important in many magnetic devices. An example is the use of electromagnets to lift steel scrap. *See also:* ELECTROMAGNET.

The highest fields obtainable in air, using superconducting coils, give B values in excess of 1 tesla, at which the energy density is approximately 4×10^5 J/m^3, or 0.4 J/cm^3. The recovery of the energy takes the form of an arc if any attempt is made to reduce the current to zero by opening a switch. Some arcing occurs across the contacts of any switch which energizes a magnetic circuit, or field, and protective measures may be necessary to prevent damage.

Comparing energy properties makes the magnetic equivalent of an electrical capacitor. The magnetic "flow" quantity, or "current," becomes $d\Phi/dt$ instead of Φ and describes the movement of poles, corresponding to the flow of charge in the electrical equivalent. Although this analogy is helpful, and more systematic (including aspects such as the modeling of eddy currents), the comparison of with electrical resistance is more usual.

Magnetic field strength

It is convenient to introduce two different measures of the magnetic field: the flux density, B, and the mmf per meter, referred to as the field strength, or field intensity, H. The field strength, H, provides a measure of the currents and other magnetic field sources, excluding those representing polarizable materials. It may also be defined in terms of the force on a unit pole.

A straight wire carrying a current I sets up a field (**Fig. 5**) whose intensity at a point at distance r is given by Eq. (13).

$$H = \frac{I}{2\pi r} \tag{13}$$

The field strength, H, like B, is a vector quantity pointing in the direction of rotation of a right-hand screw advancing in the direction of current flow. The intensity of the field is shown by the number of field lines intersecting a unit area. The straight wire provides one example of the circuital law, known as Ampère's law,

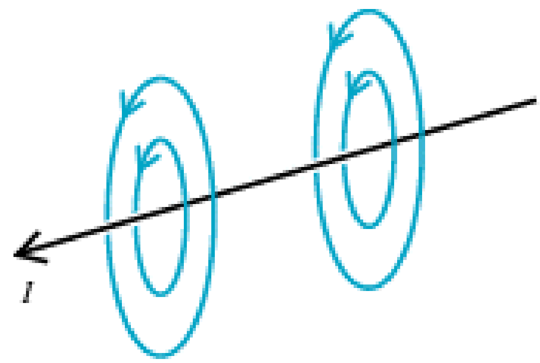

Fig. 5 Magnetic field of a straight wire.

given in vector notation by Eq. (14).

$$\oint H \cdot dl = \oint H \cos\theta \, dl = I \tag{14}$$

Here, θ is the angle between the vector **H** and the element **dl** of any closed path of summation, or integration, and H and d denote magnitudes. I is the current which links this path. Choosing a circular path, centered on a straight wire, reduces the integral to $H\,(2\pi r)$. Equation (14) cannot be applied unless a path can be found along which H is uniform, or the way in which H varies with position is known.

A long, straight, uniformly wound coil (**Fig. 6**), for example, produces a field which is uniform in the interior and zero outside. The interior magnetic field, H, points in the direction parallel to the coil axis. Applying Eq. (14) to the rectangle $pqrs$ of unit length in the axial direction shows that the only contribution is from pq, giving Eq. (15),

$$H = In \tag{15}$$

where n is the number of turns, per unit length, carrying the current, I. The magnetic field strength, H, remains the same, by definition, whether the interior of the coil is empty or is filled with ferromagnetic material of uniform properties. The interior forms part of a magnetic circuit in which In is the mmf per unit length. H is the analog of the electric field strength E, in the conductors of an electric circuit. The flux density, B, describes the effect of the field, in the sense of the voltage which is induced in a search coil by changes in time [Eq. (1)]. The ratio of H to B is the reluctance of a volume element of unit length and unit cross section in which the field is uniform. Equation (4), shows that the two quantities are related by Eq. (16). The energy density in the field [Eq.

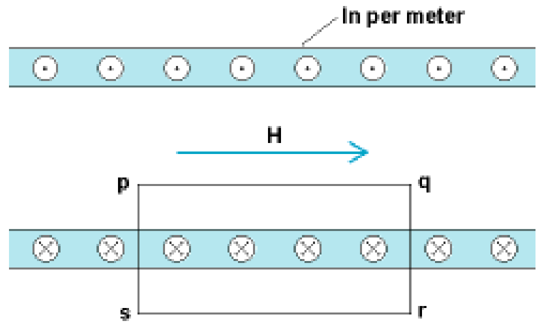

Fig. 6 **Cross section of part of a long, straight uniformly wound coil. Black circles indicate current flowing out of the plane of the page, and crosses indicate flow in the opposite direction. Rectangle *pqrs* is used to calculate magnetic field strength, *H*, within the coil.**

(11)] is given by Eq. (17).

$$B = \mu H \tag{16}$$

$$\mathfrak{E} = HB/2 \qquad \mathrm{J/m^3} \tag{17}$$

The permeability, μ, is defined by Eq. (16). The relative permeability, μ_r, of polarizable materials [Eq. (7)] is measured accordingly by subjecting a sample to a uniform field inside a long coil such as that shown in Fig. 6 and using the emf induced in a search coil wound around the specimen to observe the flux in it.

The magnetic field due to current-carrying conductors in air may also be found by dividing the circuit into elements and summing the contributions from each. Provided that I is constant, or varies only slowly in time, the field, dH, at a point, P, due to a short length, dl, of a thin wire carrying a current, I, is given by the Biot-Savart law, Eq. (18),

$$dH = \left(\frac{I\,dl}{4\pi r^2}\right)\sin\theta \tag{18}$$

where θ is the angle between the axis of the wire, in the current flow direction, and the line of length r joining P to the current element. The field lines form circles centered on the current element axis, directed according to the right-handed screw rule. The field due to a point charge, q, moving at velocity u is likewise given by Eq. (19)

$$H = \left(\frac{qu}{4\pi r^2}\right)\sin\theta \tag{19}$$

at points close enough to make the propagation delay negligible.

Magnetic flux and flux density

Magnetic flux is defined in terms of the forces exerted by the magnetic field on electric charge. The forces can be described in terms of changes in flux with time [Eq. (1)], caused either by motion relative to the source or by changes in the source current, describing the effect of charge acceleration.

Since the magnetic, or electrokinetic, energy of current flowing in parallel wires depends on their spacing, the wires are subject to forces tending to change the configuration. The force, *dF*, on an element of wire carrying a current, *i*, is given by Eq. (20),

$$dF = Bi\,dl \qquad \text{newtons} \tag{20}$$

and this provides a definition of the flux density, *B*, due to the wires which exert the force. The quantity *B* is sometimes referred to as magnetic induction. The SI unit, called the tesla, or $\mathrm{Wb/m^2}$, is the N/A-m. The flux density, *B*, equals $\mu_0 H$ in empty space, or in any material which is not magnetizable [Eq. (16)]. An example is the force which is exerted by a long straight wire on another which is parallel to it, at distance *r*. Equation (13) shows that the force, *F*, per meter length, between wires carrying currents *I* and *i* is given by Eq. (21).

$$F = \frac{\mu_0 I i}{2\pi r} = 2 \times 10^{-7}\, Ii/r \qquad \text{newtons} \tag{21}$$

when the wires carry currents *I* and *i*. *F* is accounted for by the electrokinetic interactions between the conduction charges, and describes the relativistic modification of the electric forces between them due to their motion. In consequence, *F* is small when the currents are of the order of 1 A, but is sufficient to cause damage in high-power devices, particularly when the currents are of an impulsive nature, due to fault conditions or lightning strokes. *See also:* LIGHTNING AND SURGE PROTECTION.

In general, any charge, *q*, moving at velocity *u* is subject to a force given by Eq. (22),

$$\mathbf{f} = q\,\mathbf{u} \times \mathbf{B} \qquad \text{newtons} \tag{22}$$

where $\mathbf{u} \times \mathbf{B}$ denotes the cross-product between vector quantities. That is, the magnitude of **f** depends on the sine of the angle θ between the vectors **u** and **B**, of magnitudes u and B, according to Eq. (23).

$$f = quB \sin\theta \tag{23}$$

The force on a positive charge is at right angles to the plane containing **u** and **B** and points in the direction of a right-handed screw turned from **u** to **B**.

The same force also acts in the axial direction on the conduction electrons in a wire moving in a magnetic field, and this force generates an emf in the wire. The emf in an element of wire of length dl is greatest when the wire is at right angles to the **B** vector, and the motion is at right angles to both. The emf is then given by Eq. (24).

$$\text{emf} = uB\,dl \tag{24}$$

More generally, u is the component of velocity normal to **B**, and the emf depends on the sine of the angle between **dl** and the plane containing the velocity and the **B** vectors. The sign is given by the right-handed screw rule, as applied to Eq. (23).

Since the wire moves through a distance $u\,dt$ in time dt, and B denotes flux density, the element dl “cuts through” an amount of flux given by Eq. (25).

$$d\Phi = uB\,dl\,dt \tag{25}$$

It follows that the motion of any closed loop generates a net emf given by the rate of change of flux linkage, $d\Phi/dt$, with the loop, in accordance with Eq. (1). The concept of motion relative to the field which is implied by “flux cutting” has caused much debate in applications such as homopolar devices in which B is uniform. The difficulty originates in the interpretation of velocity in Eq. (22). In general, the Lorentz force, Eq. (26),

$$\mathbf{f} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{26}$$

depends on the definition of **u** as relative to any specified inertial reference frame in which the electric field is **E**. The magnetic field sources are not necessarily stationary in this reference. They may contribute to **E**, so that the electric field is not, in general, confined to electrostatics. **E** depends on the velocity of the observer, as does **B**, since the magnetic field of a moving charge is zero in the reference in which the charge is stationary [Eq. (19)].

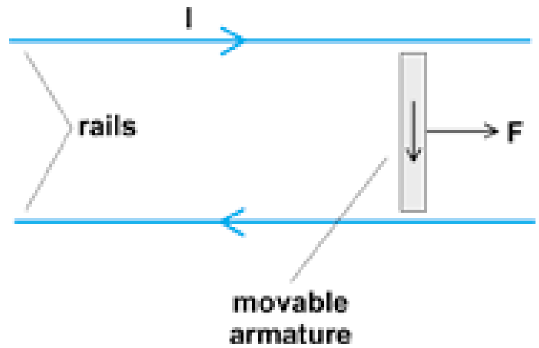

Fig. 7 **Electromagnetic railgun. Passing a large current, *I*, along the rails and through the movable armature produces force, *F*, on the armature.**

A small search coil provides a convenient way of measuring B [Eq. (2)]. Either removing or inserting it, or changing the source current, induces an emf in accordance with Eq. (1). The change in Φ is obtained by integrating the induced voltage in time as, for example, with a ballistic galvanometer, with an integrating amplifier, or by digital processing. The transverse Hall voltage which appears in a stationary current-carrying conductor as a consequence of the Lorentz forces on the moving electrons is also commonly used to measure B. *See also:* GALVANOMETER; HALL EFFECT.

Since the forces on current elements are in the direction at right angles to the element, the action and reaction forces between two elements are not in opposite directions. A force balance is obtained only between total forces on closed circuits. The electromagnetic railgun (**Fig. 7**) provides an example. Passing a large current, I, from a source to the left along the rails and through a movable armature propels the armature to the right. The source of the action is the B field of the rail currents, but these are not subject to any recoil since the forces on them are at right angles to the rails. The equilibrium is accounted for locally by attributing a state of stress to the magnetic field in the gun interior, producing an outward pressure on each of the conductors. The concept of stress, due to James Clerk Maxwell, describes the flux lines as in tension, producing the force per unit area given by Eq. (12), together with a transverse pressure of the same magnitude. The stress is commonly used to predict forces in magnetic devices, particularly when the field is calculated by numerical methods.

Equation (22) shows that any charge which moves freely in a magnetic field is subjected to a force at right angles to its motion, tending to drive it into a closed circular path. This force, quB, is balanced by the centrifugal force mu^2/r on a particle of mass m, causing it to travel in a circle whose radius is given by Eq. (27)

$$r = \frac{mu}{qB} \qquad \text{meters} \tag{27}$$

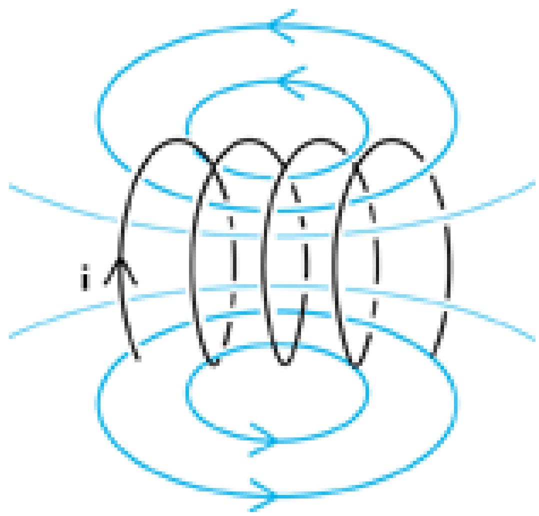

Fig. 8 ***B*** **(magnetic flux) field of a short coil.**

[with a transit time which is independent of u], unless there is a component of the velocity, u, in the direction of B. Since this component causes no interaction, the charges then tend to spiral along the flux lines. The behavior is characteristic of charged particles entering the Earth's magnetic field from outer space, and is utilized in many applications requiring the control of moving electrons, such as the magnetic focusing of electron beams. *See also:* ELECTRON MOTION IN VACUUM; VAN ALLEN RADIATION.

Magnetic flux linkage

The magnetic flux linking any closed path is obtained by counting the number of flux lines passing through any surface, s, which is bounded by the path. Stated more formally, the linkage depends on the sum given in Eq. (28),

$$\Phi = \iint B_n \, ds \qquad \text{webers} \tag{28}$$

where B_n denotes the component of B in the direction normal to the area element, ds. The rate of change of linkage gives the emf induced in any conducting wire which follows the path [Eq. (1)].

The flux linkage with a coil (**Fig. 8**) is usually calculated by assuming that each turn of the coil closes on itself, giving a flux pattern which likewise consists of a large number of separate closed loops. Each links some of the turns, so that the two cannot be separated without breaking, or "tearing," either the loop or the turn. The total linkage with the coil is then obtained by adding the contributions from each turn.

Linkage can be defined only when both the flux lines and the conductors form closed paths, so that its use depends on the absence of magnetic monopoles, since these terminate flux lines. The linkage concept cannot be applied to circuits which are not closed, such as some forms of wire antenna, and it may be difficult to apply to conductors of complex shape, as in coils that are not closely wound but form an open spiral (Fig. 8), producing a flux pattern in which the flux lines likewise form spirals. The concept of linkage attributes the induced emf to flux which may be remote from the wire in which the emf is induced, and this action-at-a-distance effect has been the source of much discussion in some applications. One such application is the magnetic Aharonov-Bohm effect, which describes the effect of the flux linkage on quantum phase. All of the actions attributed to flux linkage can be described more directly in terms of the magnetic vector potential. *See also:* AHARONOV-BOHM EFFECT.

The inductance, L, is a property of a circuit defined by the emf which is induced by changes of current in time, as in Eq. (29).

$$e = -L\, di/dt \tag{29}$$

The SI unit of inductance is the henry (H), or V-s/A. The negative sign shows that e opposes an increase in current (Lenz's law). From Eq. (1) the inductance of a coil of N turns, each linking the same flux, Φ, is given by Eq. (30),

$$L = N\Phi/i \qquad \text{henrys} \tag{30}$$

so that the henry is also the Wb/A. When different turns, or different parts of a circuit, do not link the same flux, the product $N\Phi$ is replaced by the total flux linkage, Φ, with the circuit as a whole. Applying the result to a air-cored coil of N turns whose length, d, is sufficient to give an approximately uniform field, H, in the interior [Eq. (15)] gives Eq. (31),

$$L = \mu_0 N^2 \pi r^2/d \qquad \text{henrys} \tag{31}$$

if all of the turns (Fig. 6) are circular in shape with a common radius, r. Doubling the number of turns produces a fourfold increase in L because the same current produces twice as much flux, linking twice as many turns. *See also:* INDUCTANCE.

Reducing the length brings the turns closer together and increases the inductance, since this consists of the sum of the mutual interactions between the various parts. The mutual inductance, M, between any two coils, or circuit parts, is defined by emf which is induced in one by a change of current in the other. Using 1 and 2 to distinguish between them, the emf induced in coil 1 is given by

$$e_1 = -M_{12}\, di_2/dt \tag{32a}$$

$$e_2 = -M_{21}\, di_1/dt \tag{32b}$$

Eq. (32*a*), where the sign convention is consistent with that used for *L* when the coils are connected in the series. The quantity *L* is referred to as the self-inductance. Likewise, the emf induced in coil 2 when the roles of the windings are reversed is given by Eq. (32*b*). The interaction satisfies the reciprocity condition of Eq. (33),

$$M_{21} = M_{12} \tag{33}$$

so that the suffixes may be omitted. If coil 1 consists of N_1 turns wound sufficiently closely together to link the same flux, then the mutual inductance is given by Eq. (34).

$$M = N_1\, \Phi_1/i_2 \qquad \text{henrys} \tag{34}$$

The choice of winding role may sometimes simplify the calculation. For example, a short coil of N_2 turns, each in the form of a circle of radius r_2, will link a flux which is proportional to the circles area when placed inside, and coaxial with, the coil whose self-inductance is given by Eq. (31). Applying Eq. (15) shows that the mutual inductance between the two is given by Eq. (35)

$$M = \mu_0 N_1 N_2\, \pi r_2^2/d_1 \qquad \text{henrys} \tag{35}$$

if the larger coil consists of N_1 turns wound over a length d_1. Reversing the roles makes it more difficult to calculate the mutual flux linkage when the smaller coil is too short to produce a uniform field. The mutual inductance between two coils which have approximately the same length and radius, and are sufficiently close, is given by Eq. (36).

$$M = L_1\, N_2/N_1 \tag{36}$$

This equation applies more generally to any two coils whose turns share the same flux, usually because they are linked by a common flux path of a sufficiently high permeability (Fig. 3).

Parallel wires whose length is sufficient to ignore axial variations in the field provide another important example. Current flowing in any one of them produces a magnetic field strength, H, varying inversely with the distance, r, from it [Eq. (13)]. Hence, the total flux per meter length linking two other wires, at distances r_1 and r_2 from the source, is given by Eq. (37).

$$\begin{aligned} \Phi &= \int B\, dr \\ &= (\mu_0/2\pi) i \log_e [r_2/r_1] \qquad \text{Wb/m} \end{aligned} \tag{37}$$

Dividing by the current, i, gives the mutual inductance in H/m. Any assembly of wires can be treated by superposition.

The field conditions are similar between the core and sheath of a coaxial cable consisting of a core conductor of circular cross section, with radius r_1, inside a sheath of radius r_2 carrying the return current. Equation (14) shows that the field, H, varies inversely with radius in the annular space between the two. Hence Eq. (37) gives the flux, and inductance per meter, when the frequency is sufficiently high to confine the current to the surface layers of the conductors. *See also:* COAXIAL CABLE.

Current flow paths inside the core add to the flux, giving these paths a higher inductance than those on the surface. This increases their inductive impedance, defined as the ratio of the induced emf, e, to the current. The increase is typical of internal paths, in all problems of alternating currents flowing in solid conductors, and tends to confine the currents to a surface layer whose thickness diminishes as the frequency rises. The same effect may also restrict the current to a more limited path within the layer, as when a wire carries a current, I, at high frequency close to a conducting plate (**Fig. 9**). The requirement that the inductance, and hence the magnetic flux linkage, is a minimum produces a concentration of induced current in a path immediately beneath the wire, known as the shadowing effect. The local heating may raise the temperature of the strip sufficiently to make it glow, and provides a method of controlled welding. *See also:* SKIN EFFECT (ELECTRICITY).

Magnetostatics

The term "magnetostatics" is usually interpreted as the magnet equivalent of the electrostatic interactions between electric charges. The equivalence is described most directly in terms of the magnetic pole, since the forces between poles, like those between charges, vary inversely with the square of the separation distance. Although no isolated poles, or monopoles, have yet been observed, the forces which act on both magnets and on coils are consistent with the assumption that the end surfaces are equivalent to magnetic poles.

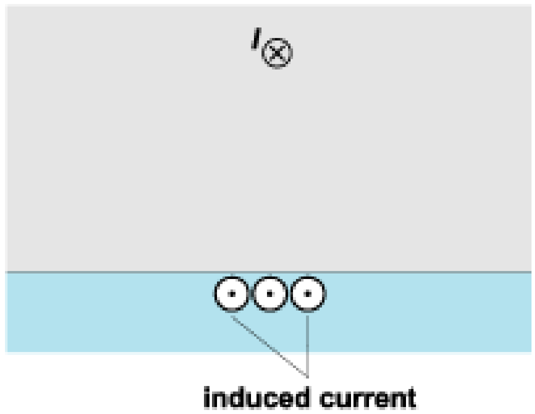

Fig. 9 Current shadowing.

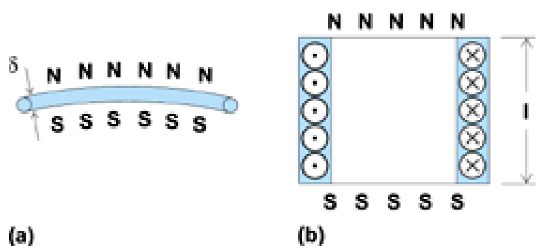

Fig. 10 Magnetic shell equivalents for (*a*) wire loop and (*b*) multiturn coil.

This follows from the magnetic shell principle. Separating two surfaces of magnetic charge of opposite sign by a small distance, δ (**Fig. 10***a*), creates a constant difference in potential, or mmf, if the charges, or poles, are uniformly distributed. A current flowing in a loop which bounds the edge of the sheets produces the same magnetic field, other than in the space between the sheets. Superposing turns shows that a uniformly wound coil of N turns of wire carrying a current, i (Fig. 10*b*), can be replaced by two sheets of magnetic charge of opposite sign, each uniformly distributed over surfaces bounded by the end turns of the coil. Both the turns and the pole sheets are shown plane in Fig. 10*b*, but in general the magnetic shell of a single turn can have any shape which is bounded by the turn, and this applies by superposition to larger coils.

The pole density, per unit area, is given by Eq. (38) for a single turn, or by Eq. (39) for a coil of N turns and length l. The total force, or torque, acting on the shell is the same as that on the loop or coil, where the force on a pole of strength q_m is given by Eq. (40).

$$\rho_m = \mu_0 i/\delta \tag{38}$$

$$\rho_m = \mu_0 iN/l \tag{39}$$

$$f = q_m H \tag{40}$$

Figure 10*b* also illustrates the two alternative models of a uniformly polarized magnet, one consisting of surface currents representing the sum of the "Ampèrean currents," and the other the surface poles. Equations (14) and (40) show that the work done in carrying a unit pole around any closed path is equal to the current I, or iN, which is linked by the path.

Replacing currents by poles is useful for computational purposes since it replaces the mmf by a magnetic scalar potential, analogous to the electrostatic potential. Both are conservative; that is, the work required to move a pole, or charge, between any two points is independent of the path taken.

Magnetic moment

The magnetic moment of a small current loop, or magnet, can be defined in terms of the torque which acts on it when placed in a magnetic flux density, B, which is sufficiently uniform in the region of the loop. Equal but opposite forces then act on opposite sides of length a of a rectangular loop of N turns (**Fig. 11**) carrying a current, i. The force is $iNBa$ [Eq. (20)], and the torque, given by Eq. (41),

$$T = iNBab \sin\theta \qquad \text{N-m} \tag{41}$$

depends on the effective distance, $b \sin\theta$, between the wires. It is proportional to the area ab, and is a maximum when the angle θ between B and the axis of the loop is 90°. A current loop of any other shape can be replaced by a set of smaller rectangles placed edge to edge, and the torques of these added to give the total on the loop. The magnetic moment, m, of any loop of area s is defined as the ratio of the maximum torque to the flux density, so that m is given by Eq. (42).

$$m = iNs \tag{42}$$

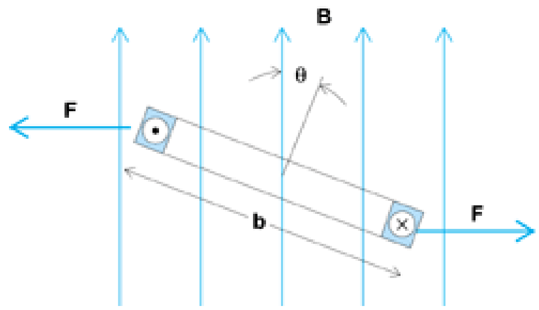

Fig. 11 **Forces acting on rectangular current loop, shown in cross section, due to the magnetic flux density *B*. Current flow direction is shown in the same way as in Fig. 6.**

This is the magnitude of a vector quantity **m** pointing in the direction of a right-handed screw turned in the direction of current flow. It gives the torque **T** in Eq. (43) in vector cross-product notation.

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \tag{43}$$

See also: TORQUE.

The coil, or magnet, may also replaced by its magnetic pole, or magnetic shell, equivalent. The torque is then accounted for by the forces on poles of strength q_m at opposite ends distance l apart, where q_m is the pole density times the area, s. Hence, from Eqs. (39) and (40), the torque is given by Eq. (44),

$$T = \mu_0 i N H s \sin\theta \tag{44}$$

and this comparison demonstrates the equivalence. The moment, m^*, may be defined by describing the torque in terms of H instead of B, giving Eq. (45).

$$m^* = \mu_0 m \tag{45}$$

An electron of charge of q_e orbiting at frequency f is the equivalent of a current $i = q_e f$, giving

$$m_0 = q_e f s \tag{46}$$

Eq. (46) for the moment, where *s* is the area of the orbit. The permissible values are determined by the quantum energy levels. The electron spin is a quantum state which can likewise be visualized as a small current loop. Atomic nuclei also possess magnetic moments. *See also:* ELECTRON SPIN; MAGNETON; NUCLEAR MOMENTS.

Magnetic polarization

Materials are described as magnetic when their response to the magnetic field controls the ratio of *B* to *H*. The behavior is accounted for by the magnetic moments produced mainly by the electron spins and orbital motions. These respond to the field and contribute to it in a process referred to as magnetic polarization. The effects are greatest in ferromagnetics and in ferrites, in which the action is described as ferrimagnetic. *See also:* FERRIMAGNETISM; FERRITE; FERROMAGNETISM.

Current-based model. The sources are the equivalent of miniature “Ampèrean currents” whose sum, in any volume element, is equivalent to a loop of current flowing along the surface of the element. The flux density, *B*, depends on the field intensity, *H*, which is defined so that its value inside a long ferromagnetic rod of uniform cross section placed inside a long coil (Fig. 6) is not affected by the rod. It is given by Eqs. (14) and (15) and is the same inside the material as in the annular gap between the rod and the coil, in accordance with Eq. (15). If the field is not sufficiently uniform, *H* can be measured by using a search coil to observe the flux density, μ_0 *H*, in the gap. The flux density inside the rod is given by Eq. (47),

$$B = \mu_r B_0 \tag{47}$$

where B_0 denotes μ_0 *H*, and μ_r is the relative permeability [Eqs. (7) and (16)]. The same flux, *B*, is obtained by replacing the material by a coil in which the current in amperes per unit length is given by Eq. (48).

$$\begin{aligned} J_s &= (B - B_0)/\mu_0 \\ &= (1 - 1/\mu_r)B/\mu_0 \end{aligned} \tag{48}$$

The magnetic moment, [Eq. (42)], of a volume element of length *dz*, due to the current J_s flowing over the surface enclosing the area, *dx dy*, is given by Eq. (49).

$$dm = (J_s\, dz)\, dx\, dy \tag{49}$$

The moment per unit volume defines the magnetic polarization, as in Eq. (50).

$$M = dm/dx\,dy\,dz \tag{50}$$

The polarization, **M**, is a vector pointing in the direction of **dm** with magnitude J_s. The surface current accounts for **B** but produces an **H**-like, or $\mathbf{B}/\mu_0$, field which is entirely different from **H** in the material. Substituting from Eq. (48) gives Eq. (51).

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \tag{51}$$

This model of the material gives the flux field, **B**, as observed by the voltage induced in a search coil wound around the specimen, and **H**, becomes an auxiliary quantity representing the difference between $\mathbf{B}/\mu_0$ and the polarization, **M**. **B** is the magnetizing field to which **M** responds. The polarization, **M**, contributes to that field, when B is greater than B_0 [Eq. (47)], since the equivalent surface current is then in the same direction as the current in the magnetizing coil.

Pole-based model. The rod may also be replaced by equivalent magnetic poles distributed over the two end surfaces (Fig. 10*b*), and this alternative likewise applies to its volume elements. Since the flux lines are continuous, by definition, the component of **B** normal to the surface is the same on both sides, and the relationship $B = \mu\ H$ requires a change in H. The change, from H_0 on the outside to H, given by Eq. (52),

$$H = H_0/\mu_r \tag{52}$$

in the interior, is accounted for by the magnetic poles. The pole density, $\rho^*{}_s$, per unit surface area is given by Eq. (53).

$$\begin{aligned} \rho_s^* &= \mu_0(H_0 - H) \\ &= (\mu_r - 1)\mu_0 H \end{aligned} \tag{53}$$

Each volume element *dx dy dz* acquires a magnetic moment given by Eq. (54),

$$dm^* = (\rho_s\,dx\,dy)\,dz \tag{54}$$

and a corresponding magnetic moment, $\mathbf{M}^*$, per unit volume. The equivalent poles are defined so as to produce the same field, **H**, as in the material, and the corresponding μ_0 **H** flux field is entirely different from **B**, which is obtained by adding $\mathbf{M}^*$ to μ_0 **H**, as in Eq. (55).

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}^* \tag{55}$$

It is B which becomes the auxiliary vector in the pole-based model, whose polarization vector, $\mathbf{M}^*$, is μ_0 **M**, in accordance with Eq. (45). If a sample of the material is placed in an air gap, the surface poles acquire an opposite sign to those on the iron surfaces adjacent to them.

Additional equivalent sources appear inside the material when the changes in permeability are not confined to the surfaces. The customary use of **H** as the "magnetizing field" follows from the historical role of the pole as the magnetic analog of electric charge, and the corresponding comparison between the treatment of magnetic materials, in terms of displaced poles, with dielectrics, whose behavior depends on the displacement of charge. Both **B** and **H** represent averages of the complex field conditions which are produced by arrays of dipoles, and are difficult to observe since local measurements require cavities whose shape determines the result. The potential functions provide more meaningful measures, in terms of the array energy, and provide a definition of the vector **B** as the differential of the magnetic vector potential, **A**, discussed below.

Magnetic hysteresis

The relationship between the flux density, B, and the field intensity, H, in ferromagnetic materials depends on the past history of magnetization. The effect is known as hysteresis. It is demonstrated by subjecting the material to a symmetrical cycle of change during which H is varied continuously between the positive and negative limits $+H_m$ and $-H_m$ (**Fig. 12**). The path that is traced by repeating the cycle a sufficient number of times is the hysteresis loop. The sequence is counterclockwise, so that B is larger when H is diminishing than when it is increasing, in the region of positive H. The flux density, B_r, which is left when H falls to zero is called the remanence, or retentivity. The magnetically "hard" materials used for permanent magnets are characterised by a high B_r, together with a high value of the field strength, $-H_c$, which is needed to reduce B to zero. The field strength, H_c, is known as the coercive force, or coercivity. Cycling the material over a reduced range in H gives the path in Fig. 12 traced by the broken line, lying inside the larger loop. The locus of the tips of such loops is known as the normal magnetization curve. The initial magnetization curve is the B-H relationship which is followed when H is progressively increased in one direction after the material has first been demagnetized ($B = H = 0$).

A magnet which forms a part of a magnetic circuit (Fig. 2) is driven to the point, P, in the part of the hysteresis loop in which B and H are in opposite directions. The mmf of the magnet of length l is Hl if the field intensity, H,

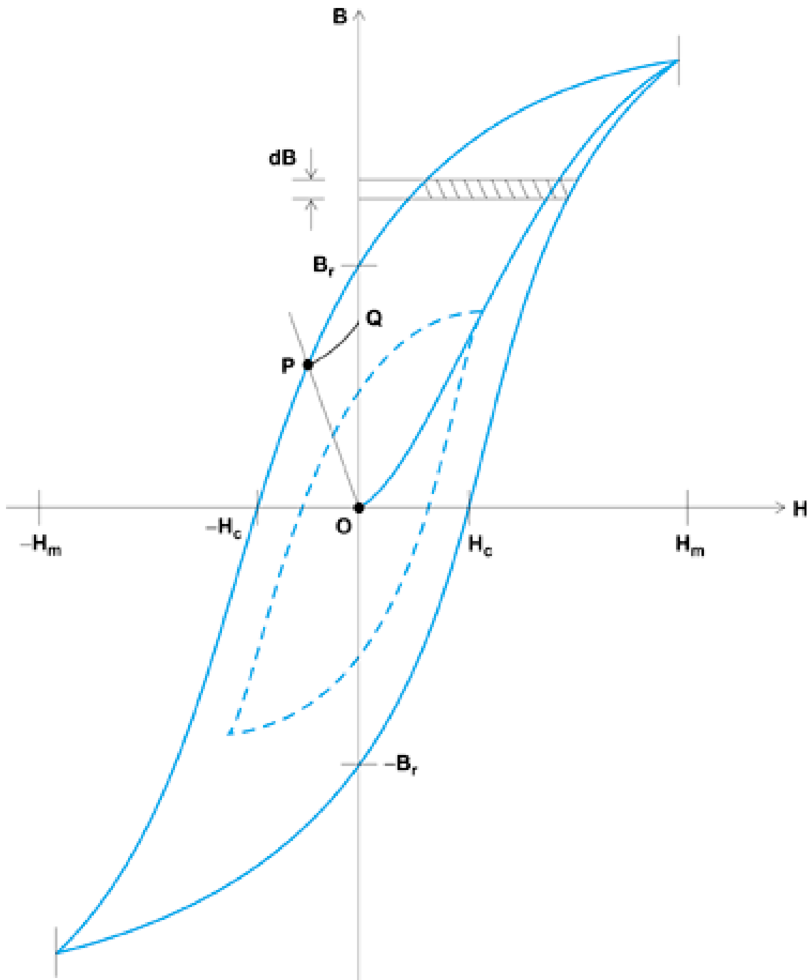

Fig. 12 **Hysteresis behavior of a ferromagnetic material.**

is the same through the volume of the material. The reluctance of the rest of the magnetic circuit produces an equal but opposite mmf in response to the flux, Φ, obtained by multiplying B by the magnet cross-sectional area, a. Hence, Eq. (56) holds.

$$Hl = \Re Ba \tag{56}$$

This approximation to the position of P can be expressed graphically by observing that the slope, B/H, of the line OP is inversely proportional to . Reducing the reluctance does not, in general, return the magnet to the point B_r, but establishes a path, PQ, known as a recoil curve, forming a part of a minor hysteresis loop.

Materials that are easily demagnetized are referred to as magnetically soft. When these are used in applications in which the supply current alternates in time, the hysteresis action changes the time phase between B and H, producing losses in the form of heat. The field strength, H, is a measure of the current, i, flowing in a single turn of suitable shape which is required to magnetize a volume element of material of unit size. Since the flux linking the coil, of unit area, is B, the voltage, e, induced in the coil [Eq. (1)] is given by dB/dt, and the energy supplied to the element during any interval of time, t, is given by Eq. (57).

$$\int ei\,dt = \int (dB/dt)H\,dt$$

$$= \int H\,dB \tag{57}$$

In the last integral, $H\,dB$ is the area of a strip of width dB drawn between the B-H curve and the $H = 0$ axis (Fig. 12), so that the energy injected is the area over the curve. When B varies linearly with H, up to the values B_m and H_m, the area forms a triangle, and the input, given by Eq. (58),

$$\int ei\,dt = H_m B_m/2 \tag{58}$$

is recovered when the line retraced is in accordance with the stored energy density in Eq. (17). Retracing the magnetization curve along a different path results in a hysteresis loop whose area is the loss, W, per unit volume per cycle. The Steinmetz coefficient, η, gives the net loss in terms of the empirical relationship of Eq. (59).

$$W = \eta(B_m)^n \tag{59}$$

The exponent n may vary between 1.5 and 2.5. The Steinmetz value of 1.6 is appropriate for many materials.

Vector potential

The use of magnetic poles provides a treatment of magnets and current-carrying coils in terms of a magnetic scalar potential, but the application is limited by the need to replace current sources by magnetic pole equivalents. The more direct description of moving charges, or currents, as field sources requires the magnetic vector potential, **A**, whose significance is illustrated by its application to the calculation of induced emfs. A magnetic field which changes in time induces an emf in an element of wire of length dl given by Eq. (60),

$$de = -(dA/dt)\,dl\,\cos\theta \tag{60}$$

where θ is the angle between the vectors **dl** and **A**. The total, *e*, for any wire, or conducting path, is obtained by summation. The quantity $N\Phi$ in Eq. (1) can therefore be replaced by the symbol Ψ, defined by Eq. (61),

$$\Psi = \int A \cos\theta \, dl \tag{61}$$

without reference to flux or flux linkage, as in Eq. (28). The integral may extend around the *N* turns of a coil by adding the contributions of each, but does not require that the path, or any part of it, closes on itself. Other advantages include the description of emf in terms of the local values of **A** at the wire, in place of flux linkages requiring the summation of remote values of *B* over surfaces bounded by the wire [Eq. (28)].

The relationship between the magnetic field and the source currents is described most directly by treating an element, *dl*, of a wire carrying a current, *i*, as the magnetic analog of electric charge. The magnitude of **A**, like the electric potential, φ, of a point charge, varies inversely with the distance, *r*, from the source in accordance with Eq. (62).

$$\mathbf{dA} = \frac{\mu_0 i \, \mathbf{dl}}{4\pi r} \tag{62}$$

dA points in the direction of current flow in the element. This is an example of the more general relationship given by Eq. (63)

$$\mathbf{A} = \boldsymbol{u}\varphi / \boldsymbol{c}^2 \tag{63}$$

between the vector potential, at point *P*, due to any group of charges moving at velocity ***u*** and the potential, φ, of the same group at *P*. Here, ***c*** is the velocity at which changes in both potentials propagate in empty space (the velocity of light). It follows that the constant μ_0 is given by Eq. (64)

$$\mu_0 = 1/\varepsilon_0 c^2 \tag{64}$$

in terms of ε_0, sometimes known as the permittivity of free space.

The mutual inductance between any two wires, as defined by Eqs. (32) and (60), is given by Eq. (65)

$$M = \Psi_2 / i_1 \tag{65}$$

in terms of the summation along wire 2 of the contributions to **A** from the current, i_1, flowing in wire 1. The calculation cannot be expressed in terms of flux linkage if wire 2 does not form a closed circuit. The two summations can be combined to give Eq. (66)

$$M = \mu_0 \int_1 \int_2 (1/r)\, dl_1\, dl_2\, \cos\theta \tag{66}$$

in terms of elements dl_1 and dl_2 at distance r and angle θ to each other. The result, due to Franz Neumann, is limited to circuits in air formed from wires, or filaments, in which the current is everywhere the same. In general, it is necessary to calculate **A** explicitly, particularly when considering the effects of induced currents and of ferromagnetic materials.

The magnetic field due to currents flowing in parallel straight wires is described by an **A** vector which points everywhere in the axial direction. Equation (63) shows that an equipotential map of the surfaces on which the magnitude **A** is constant is identical to the map of the φ equipotentials when the same wires carry charge instead of current. Hence the inductances between various conductors, when the currents are confined to the surface layers, are proportional to the reciprocal of the capacitances between the same conductors. Closing up any current-carrying wire into a loop (**Fig. 13**) likewise “curls up” its **A** vector, since Eq. (62) requires that the direction of **A** at any point, P, close to the wire is that of the nearest current. If the frequency is sufficiently low to give the same current at all points in the wire, then **A** likewise forms closed loops. Equations (28) and (61) can be expressed in the differential form of Eq. (67),

$$\mathbf{B} = \mathrm{curl}\,\mathbf{A} \tag{67}$$

and this provides an alternative definition of magnetic flux density. The “general equations of the electromagnetic field” set out by Maxwell in his *Treatise* depend on this use of **A** as the principal measure of the magnetic field, and **B** as a symbol for its differential. Maxwell’s approach is reversed in modern practice, in which **B** is treated as the source and **A** derived from it, leaving open the choice of the vector property of divergence, div **A**, or gauge. Equation (62) describes the magnetic field of a current element in terms of an **A** vector which does not necessarily close on itself, but satisfies the Lorentz condition, given by Eq. (68).

$$\mathrm{div}\,\mathbf{A} = -(1/c^2)\,\partial\varphi/\partial t \tag{68}$$

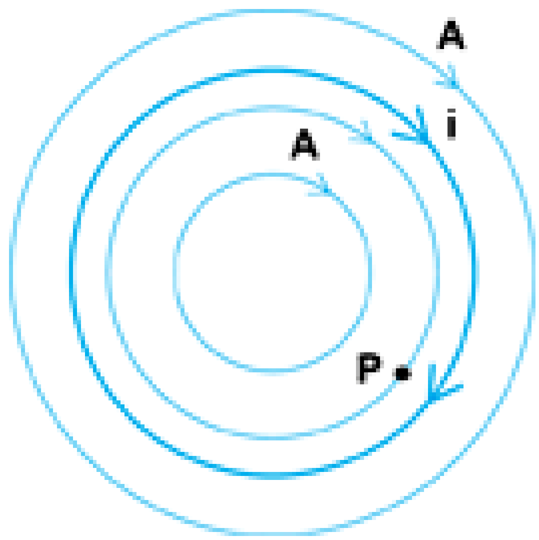

Fig. 13 **Magnetic vector potential, *A*, due to a loop of wire.**

The emf, *de*, in Eq. (60) is an example of the force given by Eq. (69),

$$\mathbf{f} = -q d\mathbf{A}/dt \tag{69}$$

which acts on any charge, q, due to a change of **A**. This provides a description of charge behavior in terms of an electrokinetic momentum, q **A**, predicting forces which tend to oppose any change. Interrupting the movement of the conduction charges in a current-carrying winding, for example, by opening a switch, causes an arc to form across the contacts due to a rise in electrical potential which is analogous to the rise in pressure when the flow of a fluid through a pipeline is suddenly blocked. The comparison illustrates the physical significance of the circuit inductance. The quantity q **A** describes the sum of the interactions between all of the moving electrons and q, and is thus a property of the system as a whole. The magnetic, or electrokinetic, energy is likewise obtained by summing the energy density, $\mathbf{J} \cdot \mathbf{A}/2$, over all of the currents, of density **J**, and it is this from which the alternative description of energy as a property of the field [Eq. (17)] is derived. The vector **A** is sufficient to describe many magnetic, or electrokinetic, effects without reference to the vector **B**. *See also:* POTENTIALS.

C. J. Carpenter

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